JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 10, No. 1, January–March 1996

Diffusion Flame over a Continuous Moving Fuel Plate Under Microgravity

M. Vedha-Nayagam*
Analex Corporation, Brook Park, Ohio 44142-1003
and
G. Fairweather†
Colorado School of Mines, Golden, Colorado 80401-1887

An analysis of the problem of a diffusion flame embedded in a laminar boundary layer developed by a continuous moving solid fuel plate in a parallel flow of oxidizer is presented. The classical Blasius-type (BT) boundary-layer combustion, and the Sakiadis-type (ST) boundary-layer combustion where the boundary layer is developed by a continuously moving plate in a quiescent environment, are special cases of this generalized formulation. Numerical results are presented for a range of the mass transfer number and for a fixed Prandtl number of 0.7. The gas-phase chemistry is assumed to be infinitely fast and a unit Lewis number assumption is also made. The results show that the flames lie closer to the surface for the ST boundary layers compared to the BT ones. The BT boundary-layer flames are more sensitive to the relative velocity differences between the freestream and the fuel feed rate compared to the ST boundary layers. Results for local burning rate and skin friction are also presented.

Nomenclature

B = mass transfer number, Eq. (12)
C_p = specific heat at constant pressure
F = dimensionless stream function
h = enthalpy
k = thermal conductivity
L = effective heat of vaporization
M_i = molecular weight of species i
$\dot{m}''$ = surface mass flux
Pr = Prandtl number
Q = heat of combustion per ν_f moles of fuel
r = stoichiometry parameter, Eq. (12)
T = temperature
u = streamwise velocity
v = transverse velocity
x = streamwise coordinate
Y_{fT} = fuel mass fraction in transferred gas
Y_i = mass fraction of species i
y = normal coordinate
α = normalized coupling function, Eq. (10)
η = similarity variable, Eq. (14)
μ = dynamic viscosity
ν = kinematic viscosity
ν_i = stoichiometric coefficients
ρ = density
ψ = stream function

Subscripts
f = fuel
o = oxidizer
w = fuel surface
∞ = ambient

Received Dec. 27, 1990; revision received June 20, 1995; accepted for publication June 23, 1995.
*Senior Engineer. Member AIAA.
†Professor and Head, Department of Mathematical and Computer Sciences.

Introduction

IN this article, a theoretical model is presented for a steady, two-dimensional, laminar diffusion flame embedded in a boundary layer produced by a continuously moving fuel plate in a parallel stream of oxidizer. Sakiadis[1] was the first to identify and analyze the boundary layer produced by a continuous sheet issuing from a slot into a quiescent fluid medium. In Sakiadis' problem, the viscous forces produced by the plate movement entrain the ambient fluid and result in a boundary layer substantially different from the classical Blasius problem of flow over a semi-infinite flat plate.[2] A detailed theoretical and experimental study of this problem was performed by Tsou et al.[3] for several Prandtl numbers. More recently, Abdulhafez[4] included an accompanying parallel freestream in the Sakiadis problem and showed that the Blasius and Sakiadis solutions are two special cases of this general problem. He also showed that the Reynolds number based on a characteristic velocity, and the velocity difference between the freestream and the plate as well as their ratio influence of the boundary-layer behavior. Chapidi and Vajravelu[5] studied the effects of blowing/suction and internal heat generation in the problem of Ref. 4.

To the authors' knowledge there has been no previous theoretical or experimental investigation of a diffusion flame in a Sakiadis-type (ST) boundary layer. The motivation for this study comes from a need to identify experimental configurations that could be used in a space-based, microgravity combustion facility to investigate basic combustion phenomena such as flame spread over solid fuels and other fire-safety related concerns. Earlier experimental studies on the combustion of solid fuels under microgravity conducted in a drop-tower included both stationary[6] and moving[7] fuel samples of fixed length. Foutch[7] simulated the effect of slow convective flow by moving a wax-coated fuel sample at a steady speed in a quiescent oxidizer environment. With the availability of a long duration microgravity environment in a space-based facility, the idea of using a continuous fuel sample that would facilitate the study of long-duration, steady phenomena as well as help develop diagnostic tools for such experiments, seems attractive. The analysis presented here is a first step in developing such a fuel-feed configuration.

Laminar diffusion flames have been studied in a variety of boundary-layer configurations in the past. For example, Pagni[8] examined diffusion flames in forced, free, mixed, and stagnation point laminar boundary layers in an effort to understand the commonality among them from the point of view of fire safety. Gravitational effects on diffusion flames in forced convection,[9] and stagnation-point[10] laminar boundary layers have also been investigated.

In formulating the general problem of boundary-layer combustion of a continuously moving fuel plate in a forced flow of oxidizer, it is assumed that the diffusion flame is anchored at the slot exit. This requires that the flame spread rate over the fuel surface U_F is greater than the fuel feed rate U_w, and that the Reynolds number based on U_w and a characteristic length is sufficiently large to ensure that the problem is steady and the boundary-layer approximation is valid. Experimentally measured spread rates under microgravity[6] show that for thin fuels at high oxygen concentrations this condition can be met. It must be noted that the previously mentioned requirements are applicable only when the freestream velocity is lower than the fuel feed velocity and the boundary layer is generated essentially by the motion of the fuel plate. When the freestream velocity dominates the flow, the condition necessary for the existence of a boundary layer is well known.[2]

Theoretical Analysis

Consider a sheet of combustible material, insulated at the bottom, issuing from a slot in a direction parallel to the flow of oxidizer at a constant velocity U_w as shown in Fig. 1. A diffusion flame is established and anchored at the origin located at the slot exit. It is assumed that U_F is greater than U_w, and that the leading edge of the diffusion flame is not pulled away from the origin by this fuel-feed motion.

The governing equations written for steady, two-dimensional, variable density, laminar boundary-layer flows, assuming unit Lewis number ($k/C_p = \rho D$), are

Mass

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

Energy

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{C_p}\frac{\partial h}{\partial y}\right) + \dot{q}''' \tag{3}$$

Species

$$\rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{C_p}\frac{\partial Y_i}{\partial y}\right) + \dot{m}_i''' \tag{4}$$

where h is given by

$$h = \int_{T_\infty}^{T} C_p \, \mathrm{d}T \tag{5}$$

$\dot{q}'''$ is the volumetric heat release rate, and $\dot{m}_i'''$ is the volumetric mass-generation rate of the ith species whose concentration measured in mass fraction is Y_i. The combustion reaction is represented by a one-step, irreversible process as follows:

$$\nu_f(\text{fuel}) + \nu_o(\text{oxidant}) \rightarrow \nu_p(\text{products}) + Q(\text{heat}) \tag{6}$$

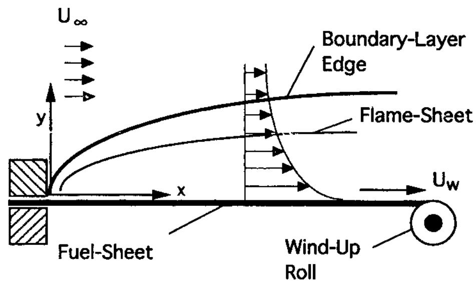

Fig. 1 Schematic of flame spread over a continuously moving fuel plate in a parallel flow of oxidizer.

The overall stoichiometry then gives the following equivalence:

$$\frac{\dot{q}'''}{Q} = -\frac{\dot{m}_f'''}{M_f \nu_f} = -\frac{\dot{m}_o'''}{M_o \nu_o} \tag{7}$$

The boundary conditions far from the fuel surface are

$$u \rightarrow U_\infty, \quad Y_o \rightarrow Y_{o\infty}, \quad Y_f \rightarrow 0, \quad \text{and} \quad T \rightarrow T_\infty, \quad \text{as} \quad y \rightarrow \infty \tag{8}$$

At the fuel surface, the no-slip condition gives $u = U_w$, and the assumption that the fuel vaporizes at a constant temperature gives $T = T_w$. Also, the oxygen concentration is zero, i.e., $Y_o = 0$, at the fuel surface in the limit of infinitely fast gas-phase chemistry. The remaining boundary conditions at the wall consist of energy and mass balance, written respectively, as

$$\rho v = \frac{k}{LC_p}\frac{\partial h}{\partial y} \quad \text{and} \quad \rho v(Y_{fT} - Y_f) = -\frac{k}{C_p}\frac{\partial Y_f}{\partial y} \quad \text{at} \quad y = 0 \tag{9}$$

where Y_{fT} is the fuel mass fraction in the wall material.

The source terms in Eqs. (3) and (4) can be eliminated using the normalized Schvab–Zeldovich variables defined as

$$\alpha_{ho} = \frac{h/L + (Y_o - Y_{o\infty})Q/(M_o\nu_o L)}{h_w/L - Y_{o\infty}Q/(M_o\nu_o L)} \tag{10a}$$

$$\alpha_{hf} = \frac{h/L + Y_f Q/(M_f \nu_f L)}{h_w/L + Y_{fw}Q/(M_f\nu_f L)} \tag{10b}$$

$$\alpha_{fo} = \frac{Y_f/(M_f\nu_f) - (Y_o - Y_{o\infty})/(M_o\nu_o)}{Y_{fw}/(M_f\nu_f) + Y_{o\infty}/(M_o\nu_o)} \tag{10c}$$

The quantity Y_{fw} is the fuel concentration at the wall, and it can be shown that it is given by (see, e.g., Ref. 8)

$$Y_{fw} = Y_{fT}(B - r)/(B + 1) \tag{11}$$

where

$$B = \left(\frac{Y_{o\infty}Q}{M_o\nu_o L}\right) - \tau \tag{12a}$$

$$r = \left(\frac{Y_{o\infty}M_f\nu_f}{Y_{fT}M_o\nu_o}\right) \tag{12b}$$

$$\tau = \left(\frac{h_w}{L}\right) \tag{12c}$$

In terms of the normalized coupling functions [Eq. (10)], Eqs. (3) and (4) combine to give

$$\rho u \frac{\partial \alpha_i}{\partial x} + \rho v \frac{\partial \alpha_i}{\partial y} = \frac{\partial}{\partial y}\left(\frac{k}{C_p}\frac{\partial \alpha_i}{\partial y}\right) \tag{13}$$

The governing nonlinear partial differential equations can be converted into ordinary differential equations using similarity variables[8] defined by

$$\rho u = \rho_\infty \frac{\partial \psi}{\partial y}, \qquad \rho v = -\rho_\infty \frac{\partial \psi}{\partial x} \tag{14a}$$

$$\psi = (U_r \nu_\infty x)^{1/2} F(\eta), \qquad \eta = \left(\frac{U_r}{\nu_\infty x}\right)^{1/2} \int_0^y \frac{\rho}{\rho_\infty}\, \mathrm{d}y \tag{14b}$$

where U_r is the reference velocity defined as

$$U_r = \begin{cases} U_w & \text{if} \quad U_w > U_\infty \\ U_\infty & \text{if} \quad U_w < U_\infty \end{cases} \tag{15}$$

The continuity equation (1) is satisfied identically by the definition of the stream function, and the momentum and coupling function equations reduce, respectively, to

$$F''' + \tfrac{1}{2} F F'' = 0 \tag{16}$$

$$\alpha_i'' + (Pr/2) F \alpha_i' = 0 \tag{17}$$

subject to the boundary conditions

$$F' = U_w/U_r, \qquad F = (2B/Pr)\alpha_{ho}', \qquad \alpha_i = 1 \quad \text{at} \quad \eta = 0 \tag{18a}$$

$$F' \rightarrow U_\infty/U_r, \; \alpha_i \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \tag{18b}$$

In Eqs. (16–18), a prime denotes differentiation with respect to η and Pr is the Prandtl number. The velocity components u and v are given by,

$$\frac{u}{U_r} = F'(\eta), \qquad \frac{v}{U_r} = (\eta F' - F)\frac{\rho_\infty}{2\rho}\left(\frac{\nu_\infty}{U_r x}\right)^{1/2} \tag{19}$$

respectively. The diffusion flame location η_f is identified as the η location where both fuel and oxygen mass fractions are zero, i.e., $Y_o = Y_f = 0$. Then, at the flame the normalized coupling function α_{fo} has the value $r/(Y_{fw}/Y_{fT} + r)$. The local fuel mass-burning rate per unit area is

$$\dot{m}'' = \rho v|_w = \frac{\rho_\infty}{2}\left(\frac{U_r \nu_\infty}{x}\right)^{1/2} [-F(0)] \tag{20}$$

The shear stress at the fuel surface expressed in dimensionless form as the local skin friction coefficient is

$$C_f = \frac{\tau_w}{\rho_\infty U_r^2/2} = 2\left(\frac{\nu_\infty}{U_r x}\right)^{1/2} F''(0) \tag{21}$$

Results and Discussions

Equations (16) and (17) subject to the boundary conditions (18) were solved numerically using the software package Colsys,[11] which implements the method of spline collocation at Gaussian points, in which the desired solution is approximated by piecewise polynomials. The effectiveness of this code in solving a variety of boundary-value problems arising in heat and mass transfer problems is well documented.[12] Two sets of calculations were performed corresponding to the ST flow for which $U_w > U_\infty$, and the Blasius-type (BT) flow where $U_w < U_\infty$ for a range of B values. Pr was fixed at 0.7 throughout the calculations. The results of these calculations are presented next.

Boundary-Layer Profiles

Figures 2 and 3 show the velocity profiles for ST and BT flows, respectively, at two different values of B. It is observed from these figures that the boundary layer becomes thicker

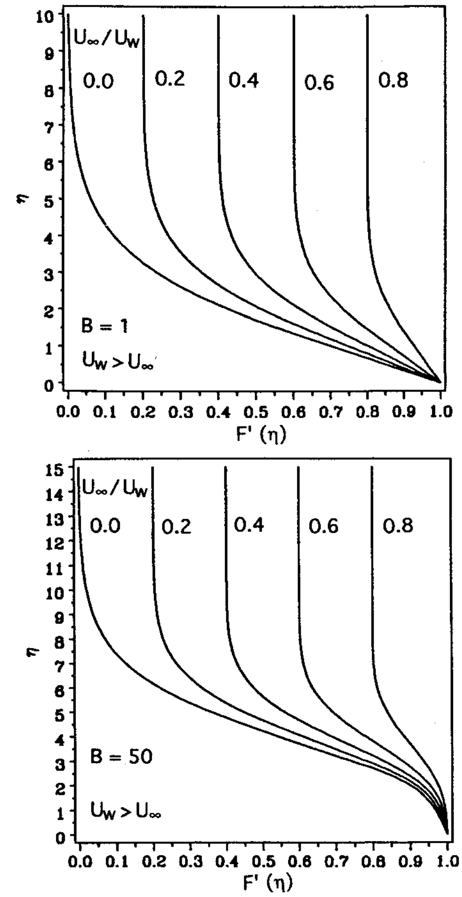

Fig. 2 Boundary-layer velocity profiles: Sakiadis-type flow.

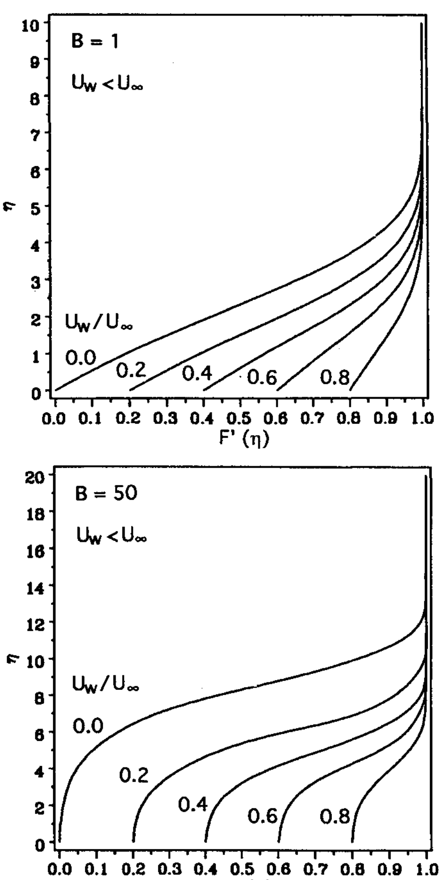

Fig. 3 Boundary-layer velocity profiles: Blasius-type flow.

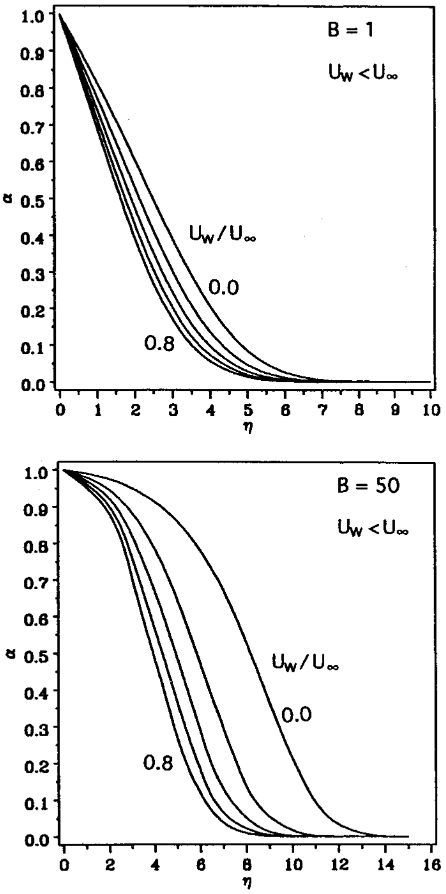

Fig. 4 Normalized coupling function profiles: Blasius-type flow.

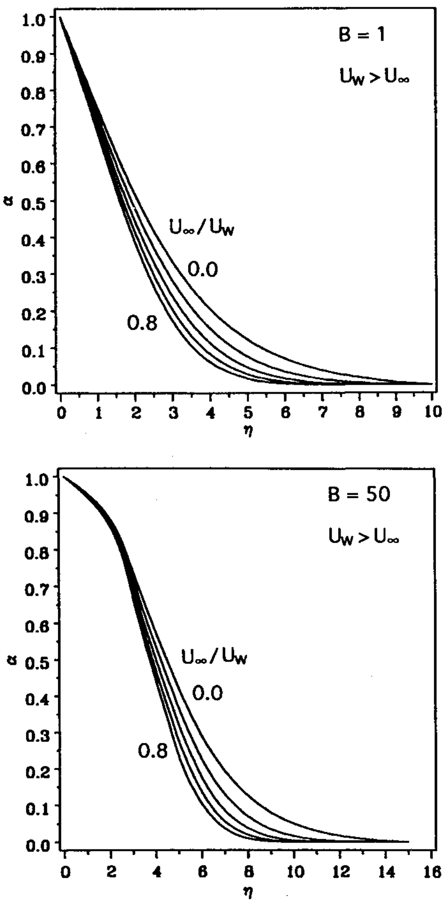

Fig. 5 Normalized coupling function profiles: Sakiadis-type flow.

as B increases, due to increased blowing at the fuel surface. For both ST and BT flows, the boundary layer becomes thinner as the velocity ratio between the freestream and the fuel plate increases. It is also worth noting that there is no velocity overshooting within the boundary layer due to combustion as is the case with normal gravity boundary-layer combustion.[9] The normalized coupling function distributions are given in Figs. 4 and 5. For a given solid fuel and environmental conditions, B and r have fixed values, and the flame is located at a constant value of α, namely, $\alpha = r/(Y_{f_w}/Y_{fT} + r)$. This implies that, in BT flow, when the velocity difference between the freestream and the fuel-feed rate $|U_w - U_\infty|$ decreases, the flame moves closer to the surface. From Figs. 4 and 5 it can also be seen that for the same fuel, ambient oxygen concentration, and velocity ratio the flame sheet is located closer to the surface for the ST boundary layer compared to the BT case. It should be noted that the relative velocity difference has a more pronounced effect for the BT flow compared to the ST flow. This is understandable because in the BT flow velocity changes are introduced close to the fuel surface where the gradients are steeper compared to the decay to the freestream. This influence is reflected in the local burning rate and wall shear stress results presented in the following sections.

Burning Rate

The local mass burning rate per unit area $\dot{m}''$ is related to $-F(0)$ as given in Eq. (20). Figure 6 shows $-F(0)$ plotted

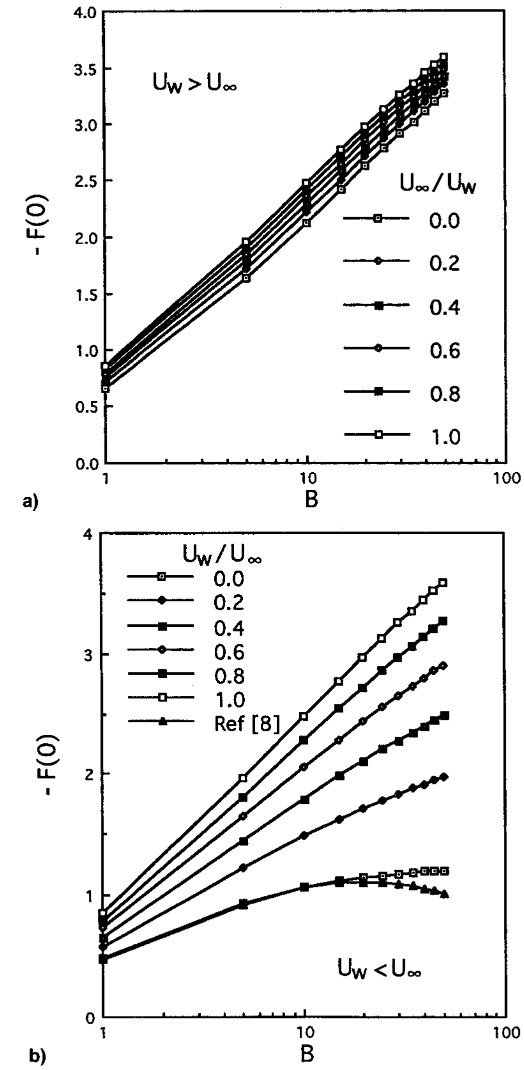

Fig. 6 Dimensionless burning rates: a) Sakiadis- and b) Blasius-type flows.

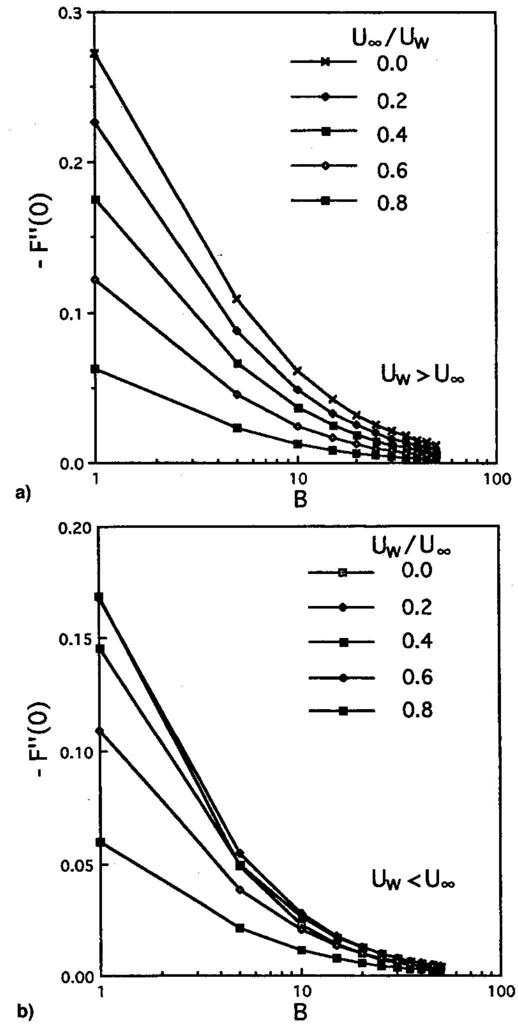

Fig. 7 Dimensionless shear stress at the fuel surface: a) Sakiadis- and b) Blasius-type flows.

against B for the ST and BT flows. For the ST flow, the surface mass transfer rate increases with both B and U_∞/U_w. The BT flows show a similar trend with B and U_w/U_∞. In both cases, as the relative velocity difference $|U_w - U_\infty|$ decreases, the boundary-layer thickness decreases and the flame gets pressed closer to the fuel surface increasing the surface mass transfer rate. However, for the BT flows, at large velocity differences between the freestream and the fuel surface the burning rate increases less rapidly with B. The results of Pagni[8] obtained for Blasius flow over a stationary fuel bed for a Pr of 0.73 shown in Fig. 6b also exhibit a similar behavior. Pagni's curve falls slightly below the present results indicating that a higher Prandtl number value with all other parameters being the same gives a lower rate of combustion.[13] As mentioned earlier, it is observed that the BT flows are more sensitive to the changes in velocity differences than the ST flows.

It is worthwhile to point out here that the results obtained for BT flows have important implications with regard to the modeling of flame spreading over solid fuels under microgravity. In a flame-fixed coordinate system, the flame spread rate, which is an unknown eigenvalue in the problem, is equivalent to the known fuel feed rate U_w in the present problem. If the spread rate is known, e.g., from experimental measurements, then the analysis presented here gives the gas-phase flowfield and the heat and mass transfer at the fuel surface. A somewhat similar approach was used by Futura et al.[14] to predict the flowfields associated with flame spread over liquid fuels.

Skin Friction

The local skin friction results are presented in Fig. 7. It can be seen that increasing B decreases skin friction for both ST and BT flows. This is again due to the fact that at high values of B there is an increased blowing at the surface, which, in turn, reduces the shear stress at the wall. Also, note that the shear stress values are higher for the ST flows compared to the BT flows for a given value of B. Again, the effect of the relative velocity difference $|U_w - U_\infty|$ on the shear stress at the wall is higher for the BT flows.

Conclusions

The problem of a continuously moving solid fuel burning in a parallel flow of oxidizer is formulated and solved for a range of values of B. The classical Blasius boundary-layer combustion and the Sakiadis combustion are special cases of this general analysis. Results show that for both BT and ST flows, the boundary layer becomes thinner and the flame moves closer to the fuel bed as the velocity difference between the freestream and the fuel feed rate $|U_w - U_\infty|$, decreases. The local skin friction and the burning rates are higher for the ST flows compared to the BT flows for the same value of B and the same velocity difference $|U_w - U_\infty|$. The flames embedded in the BT flows are strongly influenced by the velocity differences compared to the ST flows.

References

[1]Sakiadis, B. C., "Boundary-Layer Behavior on Continuous Solid Surfaces," *AIChE Journal*, Vol. 7, March 1961, pp. 26–28; Vol. 7, June 1961, pp. 221–225.

[2]Blasius, H., "Grenzschichten in Flussigkeiten mit kleiner Reibung," *Zeitschrift für Mathematik und Physik*, Vol. 56, No. 1, 1908; NACA TM 1256, Feb. 1950.

[3]Tsou, F., Sparrow, E. M., and Goldstein, R., "Flow and Heat Transfer in the Boundary Layer on a Continuous Moving Surface," *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 219–235.

[4]Abdelhafez, T. A., "Skin Friction and Heat Transfer on a Continuous Flat Surface Moving in a Parallel Free Stream," *International Journal of Heat and Mass Transfer*, Vol. 28, No. 6, 1985, pp. 1234–1237.

[5]Chappidi, P. R., and Vajravelu, K., "Boundary-Layer Behavior on a Continuous Porous Flat Surface Moving in a Parallel Free Stream," *ZAMM*, Vol. 66, No. 11, 1986, pp. 555–558.

[6]Olson, S. L., "The Effect of Microgravity on Flame Spread over a Thin Fuel," NASA TM-100195, Dec. 1987.

[7]Foutch, D. W., "Size and Shape of Solid Fuel Diffusion Flames in Very Low Speed," M.S. Thesis, Case Western Reserve Univ., Cleveland, OH, 1987.

[8]Pagni, P. J., "Diffusion Flame Analysis," *Fire Safety Journal*, Vol. 3, 1980/81, pp. 273–285.

[9]Lavid, M., and Berlad, A. L., "Gravitational Effects on Chemically Reacting Laminar Boundary Layer Flows over a Horizontal Flat Plate," *16th Symposium (International) on Combustion*, The Combustion Inst., Pittsburgh, PA, 1976, pp. 1557–1568.

[10]Foutch, D. W., and T'ien, J. S., "Extinction of a Stagnation-Point Diffusion Flame at Reduced Gravity," *AIAA Journal*, Vol. 25, No. 7, 1987, pp. 972–976.

[11]Ascher, U., Christiansen, J., and Russell, R. D., "Collocation Software for Boundary-Value ODES," *ACM Transactions on Mathematical Software*, Vol. 7, No. 2, 1981, pp. 209–229.

[12]Fairweather, G., and Vedha-Nayagam, M., "An Assessment of Numerical Software for Solving Two-Point Boundary-Value Problems Arising in Heat Transfer," *Numerical Heat Transfer*, Vol. 11, No. 3, 1987, pp. 281–293.

[13]Chen, T. N., and Toong, T. Y., "Laminar Boundary Layer Wedge Flows with Evaporation and Combustion," *Heterogeneous Combustion*, Academic, New York, 1964, pp. 643–665.

[14]Furuta, M., Humphrey, J. A. C., and Fernandez-Pello, A. C., "Prediction of Flame Spread Hydrodynamics over Liquid Fuels," *Physico Chemical Hydrodynamics*, Vol. 6, No. 4, 1985, pp. 347–372.